Some t-CI for μ.

Twenty samples of various n are selected from the normal distribution having mean 100 and standard deviation 15 (IQ distribution). Each sample is accompanied by the sample scores, n, t-score for 95% CI, sample mean, sample standard deviation, and 95% t-based CI for μ (in this case μ = 100).

You can confirm your skills by verifying some of these calculations.

Around nineteen of the twenty 95% CI should cover 100 (on average). As it happens, all twenty cover the actual population mean 100.

In[6]:=
```
s[x_] := Sqrt[Apply[Plus, (x - Mean[x]) ^ 2] / (Length[x] - 1)]
```

In[7]:=
```
s[{-1, 1}]
```

Out[7]= $\sqrt{2}$

In[30]:=
```
do[] := Module[{n = RandomInteger[{2, 31}]},
  Module[{tab = Table[RandomReal[NormalDistribution[100, 15]], {i, 1, n}]},
   {tab, MatrixForm[{{"n", n}, {"t_{n-1}", Quantile[StudentTDistribution[n - 1], 0.975]},
      {"sample mean", Mean[tab]}, {"sample standard deviation", s[tab]},
      {"95% t-basedCI", Mean[tab] +
        {-1, 1} Quantile[StudentTDistribution[n - 1], 0.975] s[tab] / Sqrt[n]} }] }]]
```

In[32]:=
```
Table[do[], {i, 1, 20}]
```

Out[32]=
```
{{{106.414, 77.7601, 92.6339, 110.257, 80.2302, 92.9829, 92.8131, 94.2179, 99.0197,
   119.244, 124.379, 106.121},
  {{n, 12}, {t_{n-1}, 2.20099}, {sample mean, 99.6727},
   {sample standard deviation, 14.2132}, {95% t-basedCI, {90.642, 108.703}}}},
 {{100.386, 101.586, 75.1226, 97.4894, 118.903},
  {{n, 5}, {t_{n-1}, 2.77645}, {sample mean, 98.6975},
   {sample standard deviation, 15.6262}, {95% t-basedCI, {79.295, 118.1}}}},
 {{75.153, 105.168, 88.2738, 109.371, 98.1758, 89.4019, 81.8247},
```

$$\left\{\begin{pmatrix} n & 7 \\ t_{n-1} & 2.44691 \\ \text{sample mean} & 92.4812 \\ \text{sample standard deviation} & 12.3805 \\ \text{95\% t-basedCI} & \{81.0311, 103.931\} \end{pmatrix}\right\},$$

$$\left\{\{122.749, 96.7415\}, \begin{pmatrix} n & 2 \\ t_{n-1} & 12.7062 \\ \text{sample mean} & 109.745 \\ \text{sample standard deviation} & 18.39 \\ \text{95\% t-basedCI} & \{-55.4821, 274.972\} \end{pmatrix}\right\},$$

$$\left\{\{92.4182, 82.428, 87.3668, 112.266, 121.954, 91.0339, 119.347, 95.707, 70.2067, 103.529,\right.$$

$$\left. 110.35, 102.937, 83.6642\}, \begin{pmatrix} n & 13 \\ t_{n-1} & 2.17881 \\ \text{sample mean} & 97.939 \\ \text{sample standard deviation} & 15.435 \\ \text{95\% t-basedCI} & \{88.6117, 107.266\} \end{pmatrix}\right\},$$

$$\left\{\{115.898, 80.7954, 92.7621, 68.6068, 104.869, 100.152, 108.444, 106.948, 93.2288, 107.777,\right.$$

$$\left. 80.8488, 99.589, 94.5037, 83.567, 95.4554, 78.9667, 104.054, 72.9941, 67.3373, 100.515,\right.$$

$$\left. 98.8302, 108.697, 109.666, 112.269\}, \begin{pmatrix} n & 24 \\ t_{n-1} & 2.06866 \\ \text{sample mean} & 95.2822 \\ \text{sample standard deviation} & 14.2019 \\ \text{95\% t-basedCI} & \{89.2852, 101.279\} \end{pmatrix}\right\},$$

$$\left\{\{106.687, 116.667, 99.6768, 89.7788, 71.4843, 105.566, 95.1321, 112.647, 91.737, 114.615,\right.$$

$$\left. 106.599, 78.441, 97.0588, 93.3314, 105.525, 85.2248, 91.6444, 103.175, 115.231, 109.639,\right.$$

$$\left. 116.631, 95.7926, 116.395, 62.5753, 97.1984, 100.819, 69.2044, 80.3315, 108.466, 103.113\},\right.$$

$$\left\{\begin{pmatrix} n & 30 \\ t_{n-1} & 2.04523 \\ \text{sample mean} & 98.0129 \\ \text{sample standard deviation} & 14.6478 \\ \text{95\% t-basedCI} & \{92.5434, 103.483\} \end{pmatrix}\right\},$$

$$\left\{\{126.308, 120.229, 105.502, 122.039, 109.217, 85.8803, 88.0762, 83.8744, 108.562,\right.$$

$$\left. 119.635, 95.2696, 114.6, 87.0112, 108.857, 113.481, 126.159, 92.7688, 92.719, 96.7225,\right.$$

$$\left. 120.135, 108.115, 103.436, 75.711, 100.855, 104.847, 74.8996, 112.977, 73.7806\},\right.$$

n	28
t_{n-1}	2.05183
sample mean	102.56
sample standard deviation	15.6669
95% t-basedCI	{96.4846, 108.635}

},

{{108.646, 104.151, 95.9393, 97.5183, 97.86, 101.026, 60.5426, 100.548, 102.846, 132.366},

n	10
t_{n-1}	2.26216
sample mean	100.144
sample standard deviation	17.4257
95% t-basedCI	{87.6787, 112.61}

},

{{100.162, 117.568, 77.3377, 101.465, 92.9737, 102.128},

n	6
t_{n-1}	2.57058
sample mean	98.6055
sample standard deviation	13.1652
95% t-basedCI	{84.7895, 112.422}

},

{{100.997, 122.172, 80.594, 106.695, 93.3091, 96.8298, 91.8187, 113.805, 109.407, 112.588, 97.0084, 80.6273, 114.259, 100.506, 120.909, 62.7173, 98.6007, 99.3361, 97.6213, 79.8653, 72.1002, 102.101, 68.996, 90.2505, 98.9674, 93.2689, 97.1384, 117.666, 120.945},

n	29
t_{n-1}	2.04841
sample mean	97.969
sample standard deviation	15.6143
95% t-basedCI	{92.0296, 103.908}

},

{{111.185, 90.5383, 81.3326, 119.737, 105.04, 121.566, 73.3624, 76.6284, 99.7515},

n	9
t_{n-1}	2.306
sample mean	97.6823
sample standard deviation	18.2025
95% t-basedCI	{83.6907, 111.674}

},

{{85.0009, 110.367, 133.484, 112.244, 102.946, 103.42, 99.5966, 110.954, 109.776, 93.1802, 105.412, 109.626, 108.67, 78.649, 120.11, 64.7997, 100.573, 128.342, 90.0592, 93.7101, 141.706, 98.3725, 59.8866, 97.6632, 141.491},

$$\left\{\begin{pmatrix} n & 25 \\ t_{n-1} & 2.0639 \\ \text{sample mean} & 104.002 \\ \text{sample standard deviation} & 20.2098 \\ \text{95\% t-basedCI} & \{95.6595, 112.344\} \end{pmatrix}\right\},$$

$$\{\{84.2284, 98.0338, 99.3206, 118.112, 98.6523, 108.65, 89.1632, 83.8518, 98.717,$$
$$95.0146, 102.511, 101.942, 106.475, 87.9991, 89.4063, 99.9557, 109.164, 118.572,$$
$$93.4303, 120.529, 97.4347\}, \left\{\begin{pmatrix} n & 21 \\ t_{n-1} & 2.08596 \\ \text{sample mean} & 100.055 \\ \text{sample standard deviation} & 10.6553 \\ \text{95\% t-basedCI} & \{95.2051, 104.906\} \end{pmatrix}\right\},$$

$$\{\{95.755, 87.5635, 94.356, 119.405, 121.462, 122.864, 96.8841, 87.4438, 131.686,$$
$$150.706, 71.3629, 97.316, 89.3454, 101.215, 112.266, 111.963, 96.6856, 115.341,$$
$$108.106, 102.107, 68.8655, 80.479, 85.6431, 87.7106, 108.159, 95.6107, 109.533\},$$

$$\left\{\begin{pmatrix} n & 27 \\ t_{n-1} & 2.05553 \\ \text{sample mean} & 101.846 \\ \text{sample standard deviation} & 18.2415 \\ \text{95\% t-basedCI} & \{94.6296, 109.062\} \end{pmatrix}\right\},$$

$$\{\{116.99, 90.8392, 106.947, 99.6923, 105.501, 108.494, 86.5387, 91.2032, 96.5872, 89.4473\},$$

$$\left\{\begin{pmatrix} n & 10 \\ t_{n-1} & 2.26216 \\ \text{sample mean} & 99.224 \\ \text{sample standard deviation} & 9.99394 \\ \text{95\% t-basedCI} & \{92.0747, 106.373\} \end{pmatrix}\right\},$$

$$\{\{92.0934, 84.3116, 69.8471, 114.335, 84.913, 85.0906, 104.3, 92.0036, 100.869, 97.4724,$$
$$95.7862, 75.3015, 95.1772, 98.9235, 101.44, 80.3223, 101.773, 105.224, 70.6271,$$
$$103.742, 105.002, 100.783, 103.449, 111.824, 120.913, 89.367, 66.9005, 100.612\},$$

$$\left\{\begin{pmatrix} n & 28 \\ t_{n-1} & 2.05183 \\ \text{sample mean} & 94.7287 \\ \text{sample standard deviation} & 13.5498 \\ \text{95\% t-basedCI} & \{89.4746, 99.9827\} \end{pmatrix}\right\},$$

$$\{\{103.089, 90.0824, 125.104, 104.408, 92.9618, 114.7, 92.8595, 83.047, 113.818,$$
$$94.3005, 99.6821, 112.87, 118.694, 107.341, 103.504, 117.514, 102.203, 111.715,$$

```
 132.198, 82.4566, 101.889}, {{n, 21}, {t_(n-1), 2.08596}, {sample mean, 104.973},
 {sample standard deviation, 13.1222}, {95% t-basedCI, {99., 110.946}}},

{{98.8642, 125.471, 85.272, 108.543, 116.938, 104.771, 89.0559, 133.869, 105.51, 111.307,
  96.3229, 107.12, 91.3131, 83.0587, 121.519, 125.978, 96.9888, 112.96, 89.845, 96.4994,
  104.605, 105.441, 101.219, 84.6774, 88.994, 86.4521, 117.007, 109.155, 103.515, 93.7336},
 {{n, 30}, {t_(n-1), 2.04523}, {sample mean, 103.2},
 {sample standard deviation, 13.4433}, {95% t-basedCI, {98.1804, 108.22}}},

{{105.575, 106.825, 94.9435, 87.0186, 103.244, 85.5006, 104.223, 97.0366, 111.445},
 {{n, 9}, {t_(n-1), 2.306}, {sample mean, 99.5346},
 {sample standard deviation, 9.00111}, {95% t-basedCI, {92.6158, 106.454}}}}
```